Data requirements for stochastic solvers

H.I. Gassmann
Dalhousie University
Halifax, Canada

Overview

- Introduction
- Algorithms
- Data structures
- Conclusions

Multistage stochastic linear program

$$
\begin{array}{lccccc}
\min & c_0 x_0 & + c_1 x_1 & + \mathrm{K} & + c_T x_T & \\
s.t. & A_{00} x_0 & & & & \sim b_0 \\
 & R_1 x_0 & & & & \Delta\ r_1 \\
 & A_{10} x_0 & + A_{11} x_1 & & & \Delta\ b_1 \\
 & \mathrm{M} & \mathrm{M} & \mathrm{O} & & \mathrm{M} \\
 & A_{T0} x_0 & + A_{T1} x_1 & + \mathrm{K} & + A_{TT} x_T & \Delta\ b_T
\end{array}
$$

$$l_0 \le x_0 \le u_0$$

$$l_t \le x_t \le u_t, t = 1, \mathrm{K}\ , T$$

Any data item with nonzero subscript may be random (including dimensions where mathematically sensible)

~ stands for arbitrary relation (≤, =, ≥)

Constraints involving random elements

$$A_{t0}x_0 + A_{t1}x_1 + \mathrm{K} + A_{tt}x_t \;\Delta\; b_t$$

- Δ means ~ with probability 1
  - or with probability at least β
  - or with expected violation at most ν

Problem classes

- Recourse problems
  - All constraints hold with probability 1
- Chance-constrained problems
  - Typically single stage
- Hybrid problems
  - Recourse problems including probabilistic constraints (VaR) or integrated chance constraints (CVaR)
  - Regulatory necessity
  - Often modelled using integer variables and/or linking constraints

Algorithms

- Direct solution of the deterministic equivalent
  - "Curse of dimensionality"
- Decomposition
  - Recognize structure
  - Repeated calls to solver with different data
  - Sampling of scenarios during algorithm
    - stochastic decomposition
    - successive approximation
    - "EVPI relaxation"
  - Scenario generator between AML and solver

Data Structures

- Often $O(10^6)$ variables and constraints
- Most compact representation possible
  - Packed matrix format is insufficient
  - Blocks corresponding to nodes in the event tree
  - Change blocks if problem dimensions are deterministic
  - $A_{stj} = A_{st0} + \Delta A_{stj}$ (addition or replacement)

Conclusions

- Stochastic extensions are difficult
- Time is right